

## AN ANALYSIS OF VARIATIONS IN CAPITAL ASSET UTILIZATION

Kwang S. Lee<sup>\*</sup>, Robert J. Hartl<sup>\*\*</sup> and Jong C. Rhim<sup>\*\*</sup>

### Abstract

This paper analyzes variations in the utilization rate of capital assets and examines the cost consequences that are often associated with increases in capital asset utilization. A generalized model is developed to measure the incremental costs of increases in capital asset utilization. The proposed model is easily adaptable for sensitivity analysis because it reduces to special forms under certain restrictive assumptions concerning planning horizon, acquisition costs, and salvage values. Further research may examine the relationship between leverage and capital asset utilization rate.

### INTRODUCTION

Investment decisions involving capital assets has long been an important subject in the financial management literature. Corporate finance theory provides robust analytical tools such as discounted cash flow analysis that are equipped with decision rules on the basis of net present value, internal rate of return and/or profitability index [2, 7, 12]. Decision science renders convenient algorithms such as variants of linear programming and goal programming.<sup>1</sup> Despite the extensive coverage afforded the subject of capital budgeting in the finance literature, there has been scant attention on the specific topic of the capital asset utilization rate decision.

Manufacturing firms frequently face decisions regarding alterations in the utilization rates of various capital assets in their employ. Whenever incremental orders are received, and excess capacity exists, these orders can be satisfied through increased capacity utilization.

An analysis of the incremental revenues associated with a planned variation in capacity utilization is a rather simple exercise. However, the incremental cost of such a decision is not so straightforward, and requires a *de facto* capital budgeting analysis. The basis for this assertion arises from the fact that an alteration in the rate of utilization can be expected to change the economic life, and thus replacement cycle, of most capital assets [10].<sup>2</sup> Hence, a business decision regarding the utilization rate of a capital asset (or assets) should be based on an incremental cost-benefit analysis that is framed in a capital budgeting context. Bierman and Smidt [2] developed a procedure to evaluate the incremental capital cost implications of a temporary increase in capacity utilization.<sup>3</sup> This procedure, while fundamentally sound, is lacking in formality, incomplete and only applicable under certain restrictive assumptions. Brealey and Myers [3] indicated this in part by pointing out that the Bierman-Smidt procedure assumes an infinite planning horizon.<sup>4</sup>

This paper analyzes variations in utilization rates of capital assets and examines the cost consequences that are often associated with these decisions. A generalized model is developed to help financial managers evaluate decisions on altering capital asset utilization rates. The proposed model reduces to special forms under certain restrictive assumptions concerning planning horizon, acquisition costs, and salvage values. An important feature of the proposed model is its adaptability to sensitivity analysis.

The remainder of this paper is structured as follows. The next section explores the economic implications of a decision to temporarily alter a capital asset's utilization rate. The following section considers the special case of an infinite planning horizon. The relationship between the proposed model and the Bierman-Smidt approach is also discussed in this section. The last section concludes the paper with suggestions for further research.

---

\*Indiana State University

\*\*University of Southern Indiana

## TEMPORARY CHANGES IN CAPITAL ASSET UTILIZATION

A company's existing stock of capital goods could be called upon to generate more output in response to a temporary increase in demand. In certain instances this could be accomplished by more intensive asset utilization during normal business hours. At other times, an asset might be more fully employed simply by extending its hours of use. Conversely, a business might be in a position to temporarily reduce the utilization rate of one or more capital goods in its possession.

Consider for a moment a special order requiring the increased utilization of existing equipment. A decision to utilize machinery and equipment in such a manner should be based upon an evaluation of incremental cash flows. The incremental revenues involved in such a decision are normally easy to identify and measure. An analyst simply needs to multiply the relevant selling price by the incremental output for each interval during the accelerated utilization period. Incremental operating costs are also reasonably straightforward to identify and estimate. In general, incremental operating costs for each time interval are found by multiplying unit variable costs by incremental output.<sup>5</sup>

When combined with the marginal tax rate and cost of capital, the incremental revenues and operating costs become the incremental contribution margin. The incremental contribution margin can be represented mathematically, as follows:

Equation 1

$$PM = \sum_{j=1}^g \frac{(r_j - v_j)(1-t)}{(1+i)^j}$$

where:

- $r$  = periodic revenue generated by the special order
- $v$  = periodic variable costs generated by the special order
- $t$  = marginal tax rate
- $i$  = cost of capital
- $g$  = accelerated utilization time frame
- $j$  = time subscript
- $PM$  = present value incremental contribution margin

In the absence of other factors, a decision criterion for special production orders would simply correspond with the following statement: Accept a special order whenever the incremental contribution margin is positive.<sup>6</sup> This is seldom the case, however, if for no other reason than the incurrence of additional capital costs. As a first approximation, an incremental capital cost results from the fact that the machinery or equipment in question will have to be replaced at an earlier date. In general, an increase in the utilization rate of a capital good will produce more wear and tear on the asset and thus shorten its remaining economic life. Furthermore, each subsequent asset in the replacement chain will have to be replaced at an earlier date as well. In the case of a decrease in demand, the opposite argument is in order. Carefully note that only existing machinery will experience a reduction in economic life. It is assumed that the replacement machines will not experience a reduction in their economic lives, since the change in utilization is only temporary. Interestingly, however, the incremental capital cost attributed to a faster replacement cycle can be partially offset by a corresponding acceleration in salvage value cash inflows, if and when they exist.

To reiterate, when a business is faced with a decision to temporarily increase the utilization rate of machinery in order to satisfy a special order, the extra workload may be expected to shorten the remaining economic life of the machinery. As a consequence, the replacement date of the capital asset in question will be moved up from the normal replacement schedule,  $d$  years in the future, to an accelerated replacement schedule,  $f$  years in the future. That is, the capital good's remaining economic life is reduced by  $(d-f)$  years. This, then, will cause all subsequent machine replacements in the planning horizon to occur  $(d-f)$  years earlier as well, even though their economic lives are unaltered. Let us represent the salvage value and acquisition cost of the existing machine as  $S_o$  and  $C_o$ , respectively. It follows that the estimated acquisition cost and salvage value of a  $j^{\text{th}}$  replacement machine are  $C_j$  and  $S_j$ , respectively. Under normal usage, the estimated economic life of a capital good is  $p$  years and there are  $k$  machine replacements during the finite planning horizon,  $n$ . Stated differently, the number of replacements,  $k$ , is expressed as  $k=(n-d)/p$ . The replacement cash flow schedule for the normal replacement cycle is shown in Exhibit 1.

**TABLE 1**  
**List of Variable Descriptions**

Variable	Description
$AEC$	Annual equivalent cost of accelerated replacement
$C_j$	Acquisition cost of the existing capital asset under normal replacement cycle
$\bar{C}_j$	Acquisition cost of the existing capital asset under accelerated replacement cycle
$\hat{C}_j$	Acquisition cost of the existing capital asset under deferred replacement cycle
$S_j$	Salvage value of the existing capital asset under normal replacement cycle
$\bar{S}_j$	Salvage value of the existing capital asset under accelerated replacement cycle
$\hat{S}_j$	Salvage value of the existing capital asset under deferred replacement cycle
$PA$	Present value of annual equivalent cost of accelerated replacement suggested by Bierman and Smidt
$PM$	Present value incremental contribution margin
$PV_d$	Present value capital cost of the normal cash flow schedule
$PV_f$	Present value capital cost of the accelerated cash flow schedule
$PV_{c1}$	Present value incremental capital cost of accelerated replacement with constant C and S
$PV_{c2}$	Present value capital cost savings of deferred replacement with constant C and S
$PV_{c3}$	Present value incremental capital cost of accelerated replacement with constant C and S for infinite planning horizon
$PV_{c4}$	Present value capital cost savings of deferred replacement with constant C and S for infinite planning horizon
$PV_e$	Present value capital cost of deferred replacement with constant C and S
$PV_{mod}$	Modified present value incremental capital cost of accelerated replacement with constant C and accelerated S

Given a cost of capital,  $i$ , the present value capital cost of the normal replacement cash flow schedule,  $PV_d$ , may be expressed as:

Equation 2

$$PV_d = \sum_{j=1}^k \frac{C_j - S_{j-1}}{(1+i)^{(j-1)p+d}} - \frac{S_k}{(1+i)^n}$$

The situation changes somewhat when the current machine's remaining economic life is reduced because of greater utilization. There will be  $k+1$  replacements in this situation, and the last one is to be terminated after only  $(d-f)$  years so as to coincide with the planning horizon.

For the accelerated replacement cycle, the cash flow schedule is depicted as in Exhibit 2. Bars are utilized here to differentiate the accelerated replacement schedule acquisition cost and salvage value from those of the normal replacement schedule.

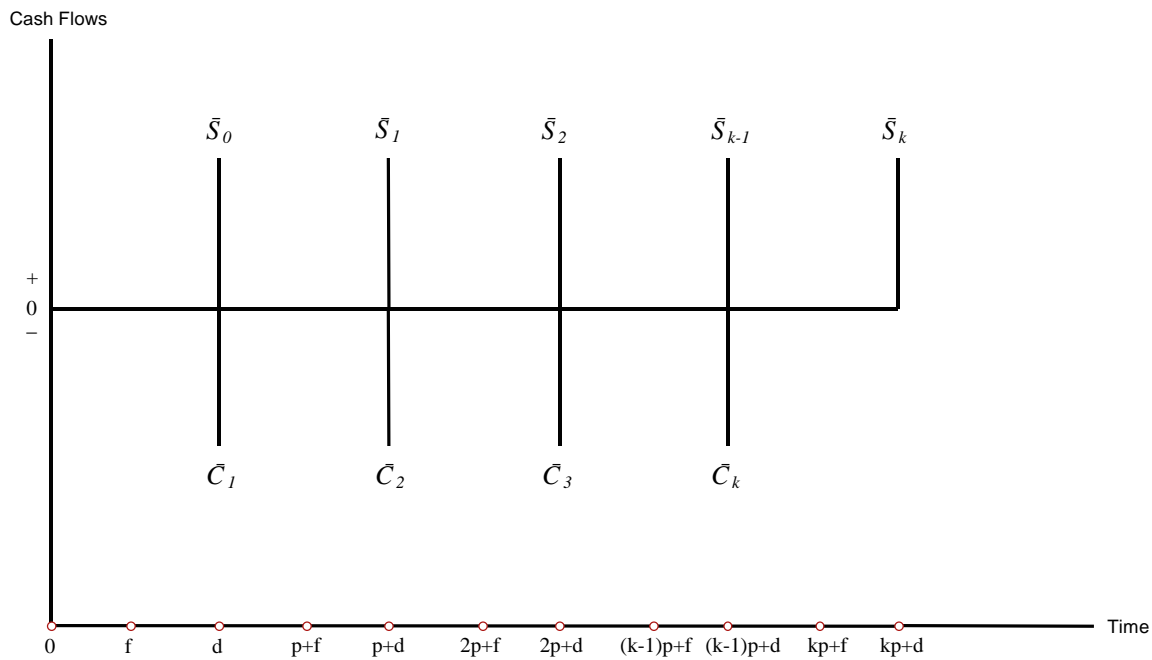
By contrast, the present value capital cost of the accelerated replacement cash flow schedule,  $PV_f$ , may be written as:

Equation 3

$$PV_f = \sum_{j=1}^{k+1} \frac{\bar{C}_j - \bar{S}_{j-1}}{(1+i)^{(j-1)p+f}} - \frac{\bar{S}_{k+1}}{(1+i)^n}$$

As stated earlier, the  $(k+1)^{th}$  replacement acquisition cost and salvage value are required for the last  $(d-f)$  years of the planning horizon, thus the summation to  $(k+1)$ .

**Exhibit 1**  
**The Cash Flows for a Normal Replacement Schedule**



The incremental capital cost of accelerated utilization,  $PV$ , may therefore be calculated as follows:

Equation 4

$$PV = PV_f - PV_d$$

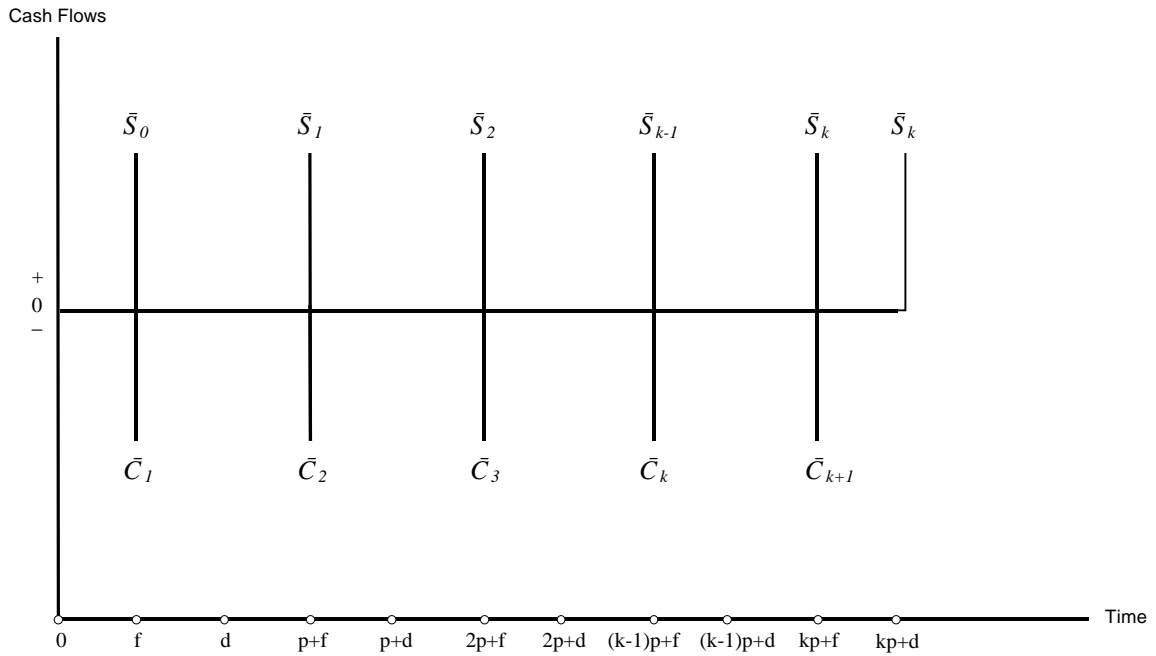
In order to complete the decision model, one simply subtracts the results in Equation 4 from those in Equation 1. In the event that more than one capital good is impacted by the decision, the various incremental capital costs must then be aggregated and compared to Equation 1.

A simpler method for computing the incremental capital cost presents itself under the assumption that:

$$C_j = \bar{C}_j = C \quad \text{for } j = 1, 2, \dots, k+1, \text{ and}$$

$$S_j = \bar{S}_j = S \quad \text{for } j = 0, 1, \dots, k.$$

**Exhibit 2**  
**The Cash Flows for Accelerated Replacement Schedule**



That is, acquisition costs and salvage values are constant. In this situation, an incremental capital cost formula may be derived from Equations 4, as follows:

Equation 5

$$PV_{cI} = (C - S) [(I + i)^{-f} - (I + i)^{-d}] \left[ \frac{I - (I + i)^{-(k+1)p}}{I - (I + i)^{-p}} \right] + (C - S_{k+1})(I + i)^{-n}$$

where:

$PV_{cI}$  = present value incremental capital cost of accelerated replacement with constant  $C$  and  $S$ .

It is clear that Equation 5 always produces a positive value since  $C > S$ ,  $C > S_{k+1}$ , and  $(I + i)^{-f} > (I + i)^{-d}$ . In other words, the present value capital cost of the accelerated replacement schedule is always greater than the present value capital cost of the normal replacement schedule. Carefully note that  $S_{k+1}$  is not necessarily equal to the other salvage values, since  $(d - f) < p$ . That is to say,  $S_{k+1}$  suffers from less wear and tear than the other salvage values. Thus,  $S_{k+1} > S$ .

**Numerical Illustration**

A wholesaler is evaluating whether or not it should increase the annual mileage from its truck fleet by 50%. The firm owns 30 trucks, all of which were purchased two years ago. Under normal usage, a truck's economic life is eight years (regular replacement in six years). With the intensified operating schedule, however, the economic life of the present truck fleet would fall to five years (accelerated replacement in three years). A new truck can be acquired for an estimated \$80,000. All salvage values are estimated to be \$15,000, with the exception of the final salvage value under the accelerated replacement schedule,  $S_{k+1}$ .  $S_{k+1}$  should be larger than the other salvage values since it occurs only three years after the final accelerated truck replacement. Assuming straight-line depreciation is a

reasonable approximation of economic depreciation,  $S_{k+1}$  is found to be  $(\$65,000 \times 5/8) + \$15,000 = \$55,625$ . The firm's planning horizon is thirty years. Accordingly, there can  $(30-6)/8 = 3$  regular replacements, or  $3 + 1 = 4$  accelerated replacements. Given a cost of capital equal to 8%, the incremental cost of increased utilization can be found with Equation 5, as follows:

$$PV_{c1} = [\$80,000 - \$15,000] [1.08^{-3} - (1.08)^{-6}] \left[ \frac{1 - (1.08)^{-(4)(8)}}{1 - (1.08)^{-8}} \right] + [\$80,000 - \$55,625] [1.08]^{-30}$$

$$PV_{c1} = \$23,585$$

Therefore, the incremental capital cost of increasing a truck's utilization rate by 50% is equal to \$23,585 per vehicle, or  $\$23,585 \times 30 = \$707,550$  for the entire fleet.

A legitimate case could be made that the salvage value,  $\bar{S}_0$ , shown in Equation 3 is not equal to the  $S_1 \dots S_k$  salvage values. Since the age of the current machine after accelerated usage is less than it would be after normal usage, the machine's salvage value under the accelerated schedule,  $\bar{S}_0$ , may be different from the normal salvage value,  $S$ . In that event, the incremental capital cost,  $PV_{c1}$ , should be modified by subtracting from  $PV_{c1}$  the present value of the accelerated salvage value minus the normal salvage value, as indicated in Equation 6.

Equation 6

$$PV_{\text{mod}} = PV_{c1} - \frac{\bar{S}_0 - S}{(1+i)^f}$$

where  $PV_{\text{mod}}$  = modified present value incremental capital cost of accelerated replacement with constant  $C$  and accelerated  $S$ .

There are also occasions when a company may have an opportunity to reduce the usage of an existing capital good (or goods), thereby extending the asset's economic life. A firm will benefit from any decision that extends the economic life of an existing machine, because of its ability to defer all of the future replacement net cash outlays ( $C_j - S_j$ ), thus saving on capital costs.

The present value capital cost of the deferred replacement cash flow schedule,  $PV_e$ , may be written as:

Equation 7

$$PV_e = \sum_{j=1}^k \frac{\hat{C}_j - \hat{S}_{j-1}}{(1+i)^{(j-1)p+e}} - \frac{\hat{S}_k}{(1+i)^n}$$

where the term  $e$  is defined as the deferred replacement date, and hats are used to differentiate the deferred case from the normal.

In general, the incremental reduction in capital costs resulting from a decision that extends the economic life of a capital good can be measured by subtracting Equation 2 from Equation 7. A simplified finite planning horizon version of Equation 7 minus Equation 2 presents itself when  $C_j = \hat{C}_j = C$  for  $j = 1, 2, \dots, k$ , and  $S_j = \hat{S}_j = S$  for  $j = 0, 1, 2, \dots, k-1$ , as follows:

Equation 8

$$PV_{c2} = PV_e - PV_d = (C - S) [(1+i)^{-e} - (1+i)^{-d}] \left[ \frac{1 - (1+i)^{-kp}}{1 - (1+i)^{-p}} \right] + \frac{S - \hat{S}_k}{(1+i)^n}$$

where  $PV_{c2}$  = present value capital cost savings of deferred replacement with constant  $C$  and  $S$ .

## INFINITE PLANNING HORIZON

The discussion up to this point has been centered around a finite planning horizon. However, in special situations an infinite planning horizon could apply. The infinite planning horizon case may be described with Equation 5 when  $n$  approaches infinity so that  $kp$  approaches  $\infty$ . Accordingly, the incremental capital cost model (i.e., Equation 5) simplifies to:

Equation 9

$$PV_{c3} = \frac{(C - S) [(I + i)^{-f} - (I + i)^{-d}]}{I - (I + i)^{-p}}$$

where  $PV_{c3}$  = present value incremental capital cost of accelerated replacement with constant  $C$  and  $S$  for an infinite planning horizon.

## Numerical Illustration

Refer back to the earlier illustration regarding the increased utilization of a fleet of trucks. Given the same  $C$ ,  $S$ ,  $i$ ,  $d$ ,  $f$  and  $p$ , and allowing the planning horizon ( $n$ ) to go to infinity, the incremental cost of greater truck utilization is determined as:

$$PV_{c3} = [\$80,000 - \$15,000] [1.08^{-3} - (1.08)^{-6}] \left[ \frac{1 - (1.08)^{-(4)(8)}}{(1.08)^{-8} - 1} \right]$$

$$PV_{c3} = \$23,131$$

In this instance, the incremental cost of increased utilization is \$23,131 per truck, or  $\$23,131 \times 30 = \$693,930$  for the entire fleet. Interestingly, the incremental cost associated with the infinite planning horizon, or \$23,131, is less than that associated with a finite planning horizon, or \$23,585. The incremental capital cost drops sharply as the number of replacements increases from 0 to 5 and then asymptotically converges to the infinite planning horizon capital cost.

Of significance in this model is the fact that, given a capital good's acquisition cost, salvage value and the cost of capital, the incremental capital cost of increased utilization is dependent on the existing machine's accelerated replacement date,  $d$ , regular replacement date,  $f$ , and the normal economic life of the machine,  $p$ .<sup>7</sup> One may ignore the number of successive machine replacements, or  $k$ .

Likewise, an alteration in Equation 8 is called for under conditions of an infinite planning horizon, thus:

Equation 10

$$PV_{c4} = \frac{(C - S)[(I + i)^{-e} - (I + i)^{-d}]}{I - (I + i)^{-p}}$$

where  $PV_{c4}$  = present value capital cost savings of deferred replacement with constant  $C$  and  $S$  for an infinite planning horizon.

Bierman and Smidt [2] intuitively addressed the incremental capital cost associated with a temporary increase in capital good utilization rates. As they pointed out, the use of excess capacity incurs relevant incremental costs. To derive this cost, they first suggest finding the annual equivalent cost of the asset,  $AEC$ , as shown in Equation 11.

Equation 11

$$AEC = \frac{iC}{I - (I + i)^{-p}}$$

Bierman-Smidt then proceed to calculate the present value of the annual equivalent costs ( $PA$ ) for those years that are lost by the additional usage. Their procedure may be formalized, as follows:

Equation 12

$$PA = AEC \sum_{j=1}^{d-f} (I + i)^{-(f+j)}$$

However, Bierman-Smidt did not consider salvage values in their analysis. If the salvage values are negligible so that  $S=0$ , Equation 9 is equivalent to the Bierman-Smidt procedure, as shown below.

Substituting Equation 11 into Equation 12, the Bierman-Smidt procedure may be expressed as:

Equation 13

$$PA = \frac{iC}{1 - (I+i)^{-P}} \sum_{j=1}^{d-f} (I+i)^{-(f+j)}$$

Furthermore,

Equation 14

$$\sum_{j=1}^{d-f} (I+i)^{-(f+j)} = \frac{1}{i} [(I+i)^{-f} - (I+i)^{-d}]$$

Substituting Equation 14 into Equation 13 provides a result which is identical to Equation 9 with  $S=0$ . Importantly, therefore, the Bierman-Smidt methodology implies an infinite planning horizon.

More to the point, the Bierman-Smidt procedure can be applied only under the following three simplifying assumptions: (i) the planning horizon is infinite, (ii)  $C_j = \hat{C}_j = C$  for all  $j$ , and (iii) salvage values are zero. However, the requirement of a constant acquisition cost may prove too restrictive in many situations. Salvage values could also be significant. And, an unwarranted assumption of an infinite planning horizon can misstate the incremental capital cost of increased utilization. This last point was also singled out by Brealey and Myers [3]. Finally, the Bierman-Smidt procedure would not appear to be intuitively appealing to financial management practitioners, nor does it lend itself well to sensitivity analysis.

## SUMMARY

Many capital goods can be utilized at varying degrees of capacity. Such variations in capacity utilization can be the result of temporary or permanent changes in product demand. The most interesting aspect of these situations concerns the effect that capacity utilization can have on an existing asset's remaining economic life and the cost ramifications. This paper has attempted to explain the rationale behind this aspect of capacity utilization and introduced as well a methodology to measure the economic impact. Several models were developed to account for differences in planning horizon (finite and infinite) and direction of altered utilization (accelerated and decelerated). In each and every case, the models are conducive to sensitivity analysis. Further research may examine the endurance of changes in capacity utilization. While this study does not examine the comparison between temporary and permanent changes in capacity utilization, results and implications will be different depending on the endurance of change. The relationship between leverage and capital asset utilization is another issue worth considering in future research.

## ENDNOTES

1. Typically the application of capital budgeting situations with linear programming is a maximization problem of net present values subject to constraints [4, 5, 11]. Binary or integer programming is commonly applied [1, 9]. Goal programming is a constrained minimization problem that can be applied for multiple objective functions [6, 8].
2. Economic life is defined here as the number of years an asset is in use until the present value cost of retention exceeds the present value cost of replacement.
3. See Bierman and Smidt [2], Chapters 7 and 12.
4. See Brealey and Myers [3], Chapter 6.
5. Unit variable costs are primarily a function of direct expenditures for such standard items as labor, materials, maintenance and energy. Of course, adjustments in some or all of these cost elements may well be necessary due to the fact that the capital goods in question will more than likely be employed in an output range associated with diminishing returns. Overtime, work-shift differentials and economies of scale could all come into play here as well.

6. This criterion would have to be slightly modified in the event that working capital is affected by the decision.
7. It is noteworthy that both the finite and infinite planning horizon incremental capital cost models are extremely well suited for sensitivity analysis. Furthermore, it is necessary that all acquisition costs and salvage values be adjusted for taxes so as to correspond with the incremental contribution margin, which is on an after-tax basis. To be more specific, acquisition costs must be adjusted downward for investment tax credits, when and if they apply. By the same token, salvage values should be adjusted downward for capital gain tax payments and upward for capital loss tax savings.

## REFERENCES

- [1] Balas, E., "An Additive Algorithm for Solving Linear Programs with Zero-One Variables," *Operations Research*, July/August 1965, pp. 517-549.
  - [2] Bierman, H. Jr. and S. Smidt, *The Capital Budgeting Decision: Economic Analysis of Investment Projects*, Upper Saddle River, New Jersey: Prentice-Hall, 1993, 8th ed.
  - [3] Brealey, R. and S. Myers, *Principles of Corporate Finance*, New York: McGraw-Hill, Inc., 1996, 5th ed.
  - [4] Carleton, W.T., "Linear Programming and Capital Budgeting Models," *Journal of Finance*, December 1969, pp. 825-833.
  - [5] De, P.K., D. Acharya and K.C. Sahu, "A Chance-Constrained Goal Programming Model for Capital Budgeting," *Journal of the Operational Research Society*, Vol. 33, No. 4, 1982, pp. 635-638.
  - [6] Hawkins, C.A. and R.A. Adams, "A Goal Programming Model for Capital Budgeting," *Financial Management*, Fall 1974, pp. 52-57.
  - [7] Kaufman, M., ed. *The Capital Budgeting Handbook*, Homewood, Illinois: Irwin, 1986.
  - [8] Lawrence, K.D. and G.R. Reeves, "A Zero-One Goal Programming Model for Capital Budgeting in A Property and Liability Insurance Company," *Computers & Operations Research*, Vol. 9, No. 4, 1982, pp. 303-309.
  - [9] McLaughlin, R. and R. Taggart, Jr., "The Opportunity Cost of Using Excess Capacity," *Financial Management*, Summer 1992, pp. 12-23.
  - [10] Pindyck, R., "Irreversible Investment, Capacity Choice and the Value of the Firm," *American Economic Review*, December 1988, pp. 969-985.
  - [11] Weingartner, H.M., *Mathematical Programming and the Analysis of Capital Budgeting Problems*, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963.
  - [12] Woods, J. and M. Randall, "The Net Present Value of Future Investment Opportunities: Its Impact on Shareholder Wealth and Implications for Capital Budgeting Theory," *Financial Management*.
-