

RATE OF RETURN ON INVESTMENTS: A NEW EVALUATION PROCEDURE

Kashi Nath Tiwari*

Abstract

This paper introduces a new procedure to evaluate the rate of return by comparing the reinvestment values of asset-prices and asset-earnings for the entire holding-period. During the investment horizon if market rates, on average, remain constant, then the rate of return (ROR) will be zero; if they rise, the ROR will be negative; and if they fall, the ROR will be positive. When market rates rise (fall), then the time-value of the asset-price-paid increases (decreases) by a larger (smaller) magnitude than the asset-earnings-received, thereby making the ROR negative (positive). These results hold true irrespective of whether the holding period is greater than, equal to, or less than duration; further, it is in stark contrast with the conventional-results, where the ROR is dependent on the length of the holding period in relation to duration. Within the purview of the proposed-model, if the bond is held to maturity, then under rising (falling) market rate conditions the ROR will be negative (positive); while in the conventional models, opposite conclusions are drawn such that under rising (falling) market rate conditions, the yield-adjusted ROR will be positive (negative).

INTRODUCTION

The rate of return model introduced in this paper evaluates the reinvestment value of the ratio of asset-earnings and asset-price such that the ROR is zero when expectations with regard to the future values of market interest rates are, on average, realized. If the actual rates, on average, exceed their expected levels, then the ROR is negative and vice versa, irrespective of the length of the investment horizon. In the conventional models, the rate of return is dependent on the length of the holding-period in relation to duration and the level of market rate movements. It does not matter at what point in time (initial, intermediate, or maturity) the investment returns are evaluated, the rate of return is shown to be zero under constant market rate conditions, negative under rising market rate conditions, and positive under falling market rate conditions. While evaluating the investment returns, this paper includes all the parametric changes that take place during the investment horizon. It is easier for investors to evaluate their financial status under the proposed procedure --- in net have investors gained or lost at time n relative to time $n-k$. The modeling procedure for the new-ROR and its central theme and contributions are as follows:

1. The new-ROR of this paper is, in fact, the real rate of return in that it measures the changes in the purchasing power of the initial investments — how much more or less goods and services can be bought at time H with the money invested at time 0 . Under falling (rising) market rate conditions, the new-ROR correctly informs the investors that it is time to be happy (sad) since the real value of their asset is appreciating (depreciating), while the old-ROR misinforms the investors by drawing opposite conclusions. Thus when new-ROR indicates that the investors should be happy, the old-ROR indicates that they should be sad, and vice-versa. The results of this paper will have significant policy implications for the investment world of financial companies and their clientele. When should investors be happy and when should they be sad? When should investors acquire an asset and when should they dispose off an asset?
2. For simplicity, the author uses the term “new-ROR” or “modified-ROR” to describe the rate of return that he introduces in this paper, while he refers to the conventional one as the old-ROR. Similarly,

*Kennesaw State University

this paper's model is referred to as the "new-model" of ROR, while the conventional models are referred to as the old-model.

3. To make the old-ROR directly comparable to the new-ROR, the author subtracts the expected-old-ROR from the realized-old-ROR and refers to it as the adjusted-old-ROR* (or, old-ROR*). Thus the expected-old-ROR is used as a proxy for the yield-to-maturity (Y).
4. The author compares the reinvestment-value of the purchase-price with the reinvestment value of the earnings to determine the new rate of return for the entire holding period. The conventional models do not use the time-value of the purchase-price, instead they use the initial purchase-price. This constitutes the fundamental difference between the new-model and the old-model, leading to diametrically opposite results, conclusions, and recommendations.
5. The time-valued asset-prices and asset-earnings become instant-prices paid and instant-earnings received for the entire holding period, and hence the ratio of the value-received and value-paid need not be subjected to geometric-averaging, which is in contrast to the conventional models, where such an averaging is a necessity. The paper also considers geometric-averaging of the ratio by treating the value-received and value-paid as the total revenues and total costs respectively for the entire holding period. The paper's results are valid irrespective of whether or not the ratio of the value-received and value-paid is subjected to geometric-averaging.
6. When market rates rise (fall), then the time-value of the asset-price-paid increases (decreases) by a larger (smaller) magnitude than the asset-earnings-received, thereby making the ROR negative (positive).
7. Stark Contrast: At maturity, the new-ROR will be positive, negative, or zero depending upon whether the market rates, on average, fall, rise, or remain constant respectively over the investment horizon. The opposite is true with respect to the adjusted-old-ROR*. That is, at maturity, the old-ROR* will be negative, positive, or zero depending upon whether the market rates, on average, fall, rise, or remain constant respectively over the investment horizon. The unadjusted-old-ROR, on the other hand, will be less than Y (yield-to-maturity), greater than Y, or equal to Y depending upon whether the market rates, on average, fall, rise, or remain constant respectively over the investment horizon.
8. If, on average, the realized market-rates are equal to their expected level, then the value of the new-ROR will be zero and so will be the value of yield-adjusted-old-ROR*.
9. If the asset is held till maturity and if, on average, realized rates are greater than their expected levels, then the value of the new-ROR will be *negative*, while that of the old-ROR will be *positive*. Alternatively, if the market rates, on average, rise then the new-ROR will be negative, while the adjusted-old-ROR* will be positive when the asset is held till maturity. In other words, when market rates rise, then the old-model informs to investors that their investment is appreciating in value, while the new-model correctly informs the investors that their investment is, in fact, depreciating. When market rates rise, investors ought to be happy following the old-model, while they, in fact, ought to be unhappy if the ROR-analysis is done in the context of the new-model. This has important implications for investment recommendations to the clientele.
10. If the asset is held till maturity and if, on average, realized rates are less than their expected levels, then the value of the new-ROR will be *positive*, while that of the old-ROR will be *negative*. Alternatively, if the market rates, on average, fall then the new-ROR will be positive, while the adjusted-old-ROR* will be negative when the asset is held till maturity. In other words, when market rates rise, then the old-model informs to investors that their investment is depreciating in value, while the new-model correctly informs the investors that their investment is, in fact, appreciating. When market rates fall, investors ought to be unhappy following the old-model, while they, in fact, ought to be happy if the ROR-analysis is done in the context of the new-model. This has important implications for investment recommendations to the clientele.
11. Irrespective of the holding (H) period relative to duration (D), if market rates, on average, rise, then the new-ROR will be always negative; it does not matter whether the holding period is equal to, less than, or greater than duration. The new-model recommends that under rising market-rate conditions, the investors should dispose off the asset. However, if the geometric-averaging of the ratio of the value-received and value-paid is accepted, then the negative-value of the new-ROR diminishes in value as the holding-period is increased; meaning thereby that the investors losses will dwindle as the

- holding-period is increased, but their losses can never be zero or positive. Under conventional models, if market rates rise, then $ROR=0$ for $H=D$, $ROR>0$ for $H>D$, and $ROR<0$ for $H<D$.
12. Irrespective of the holding (H) period relative to duration (D), if market rates, on average, fall, then the new-ROR will be always positive; it does not matter whether the holding period is equal to, less than, or greater than duration. The new-model recommends that under falling market-rate conditions, the investors should continue to hold the asset. However, if the geometric-averaging of the ratio of the value-received and value-paid is accepted, then the positive-value of the new-ROR diminishes in value as the holding-period is increased; meaning thereby that the investors gains will dwindle as the holding-period is increased, but their gains can never be zero or negative. Under conventional models, if market rates fall, then $ROR=0$ for $H=D$, $ROR>0$ for $H<D$, and $ROR<0$ for $H>D$.
 13. The proposed model attempts to make the financial language (concept of ROR) more comprehensible and accessible to the general public, rather than confining it to only a handful of sophisticated investors as it appears to be the case with the conventional-ROR. The model informs the investors that if they pay \$x at time h and receive \$x in return also at time h on an asset, then their ROR will be zero; if the price exceeds the earnings, the ROR will be negative; and if the price is less than the earnings, the ROR will be positive. The proposed model candidly informs the general public where exactly they stand in the marketplace - whether their investments are appreciating, depreciating, or remaining constant in value. That is, If investors pay \$100 at time h and receive \$100 in return also at time h, then their rate of return will be zero; if they pay only \$90 to receive \$100, then their rate of return will be positive; and if they pay \$110 to receive only \$100, then their rate of return will be negative.
 14. Appreciation and Depreciation: In the ensuing analysis, a positive rate of return is taken as to mean an appreciation in investment value, while a negative rate of return is taken as to mean a depreciation in investment value.
 15. In the conventional models, the immunization occurs at the point of duration ($ROR=Y$ or equivalently, yield-adjusted-old-ROR* equals zero). However, within the context of the new-model, the usual duration-immunization principle appears to hold little validity since the new-ROR does not equal the yield-to maturity at the point of duration.
 16. The conventional procedure excludes the impact of inflation rate on the rate of return, while the proposed procedure accounts for the inflation rate and other parametric changes. It is easier for investors to evaluate the level of their financial status under the proposed procedure --- in net have they gained or lost at time n relative to time n-k.
 17. A survey of individuals indicates an overwhelming preference for the new-ROR due to the ease of information and clarity.
 18. In paying tribute to the outstanding scholars of the conventional models, the author recommends that the subsequent evaluation of the real and financial investments be done within the purview of the new-ROR introduced in this paper.

NOTATION AND DEFINITION

In the ensuing analysis, the following notations will be used:

N	= maturity of the asset
H	= investment horizon or holding period
Y_i	= one-period expected market rate applicable for period $i+1$, where $i = 0, 1, 2, 3, \dots, n$
aY_i	= one-period actual/realized market rate applicable for period $i+1$, where $i = 0, 1, 2, 3, \dots, n$
$P_0 = PV$	= asset price at time 0, (present value of future income stream)
P_n	= at time n, the time-value of the original price-paid, P_0
P_h	= at time h, the time-value of the original price-paid, P_0
V_n	= at time n, the value of the asset earnings

V_h	=	at time h, the value of the asset earnings
${}^e P_n$	=	at time n, the <i>expected (e)</i> time-value of the original price-paid, P_0
${}^a P_n$	=	at time n, the <i>actual (a) or realized</i> time-value of the original price-paid, P_0
${}^e V_n$	=	expected (e) time-value of asset earnings in n-periods
${}^a V_n$	=	<i>Actual (a) or Realized</i> time-value of asset earnings at time n
${}^e R_c$	=	expected (e) rate of return under <i>conventional (c)</i> models
${}^a R_c$	=	realized rate of return under <i>conventional (c)</i> models
${}^e R_m$	=	expected (e) rate of return under <i>modified (m)</i> model of this paper
${}^a R_m$	=	realized rate of return under <i>modified (m)</i> model of this paper
${}^e R_{ci}$	=	at time i, expected (e) rate of return under <i>conventional (c)</i> models
${}^a R_{ci}$	=	at time i, realized rate of return under <i>conventional (c)</i> models
${}^e R_{c-y,i} = {}^e R_c - {}^e R_c = 0$	=	at time i, yield-adjusted expected ROR under conventional models
${}^a R_{c-y,i} = {}^a R_c - {}^e R_c$	=	at time i, yield-adjusted actual ROR under conventional models
${}^e R_{mi}$	=	at time i, expected (e) rate of return under the <i>modified (m)</i> model of this paper
${}^a R_{mi}$	=	at time i, realized rate of return under the <i>modified (m)</i> model of this paper
$R_{gm,i}$	=	at time i, the rate of return under the modified model by taking i^{th} root (geometric mean)

THE THEORETICAL MODEL AND ANALYSIS

The expected rate of return, ${}^e R_c$, and the realized rate of return, ${}^a R_c$, under the conventional model can be defined as:

$${}^e R_c = [{}^e V_n / P_0]^{1/N} - 1$$

$${}^a R_c = [{}^a V_n / P_0]^{1/N} - 1$$

The expected rate of return, ${}^e R_m$, and the realized rate of return, ${}^a R_m$, under the modified model can be defined as:

$${}^e R_m = [{}^e V_n / {}^e P_n] - 1$$

$${}^a R_m = [{}^a V_n / {}^a P_n] - 1$$

Uniqueness Of The New-ROR:

1. Note the fact that in the new-ROR formulae the ratios, $[{}^e V_n / {}^e P_n]$ and $[{}^a V_n / {}^a P_n]$, are not subjected to geometric-averaging (they are not n^{th} rooted), while in the conventional models the corresponding ratios are subjected to geometric-averaging. The reason being that in the modified formula, both denominator and numerator have been time-valued up to period n, while in the conventional models only the numerator is time-valued up to period n, while the denominator is time-valued only up to period 0. Under conventional framework, the price, P_0 , is paid at time 0 and the revenues are received n-period later, and hence the convention of geometric-averaging (n^{th} rooting). However, in the proposed model, the future-value of the price-paid is compared with the future-value of the investment earnings at time n, which means, *de facto*, the price (P_n) is paid at time n and the revenue (V_n) is received at time n, and hence there is no need to take the geometric-average (n^{th} root) of the ratio, V_n / P_n . The rate of return derived without taking the geometric average of the ratio, V_n / P_n , is denoted by R_{mi} in this paper. This ratio is determined to be time-neutral in the ensuing analysis.
2. If P_n , and V_n are not considered as the instant price-paid and instant revenue-received at time n, but rather P_n is considered as the total price-paid ($P_0 + \text{time value}$) in n-periods and V_n is considered as the total earnings-received in n-periods, then a geometric-average of the ratio, V_n / P_n , will produce a per period (annual) rate of return (plus one). The rate of return derived by taking the geometric average of the ratio, V_n / P_n , is denoted by $R_{gm,i}$ in this paper.

3. This paper's R_{mi} (modified ROR) be compared with the R_{ci} (conventional ROR), for investment-return evaluation purposes.
4. This paper's results and recommendations that contrast those of the conventional-models' are valid for both R_{mi} and $R_{gm,i}$. The difference between this paper's two rates, R_{mi} and $R_{gm,i}$ is only a matter of degree rather than the direction (positive or negative). The conventional R_{ci} , fails to provide a candid investment-return information to investors, while R_{mi} and $R_{gm,i}$ provide a direct and precise evaluation of investment returns.
5. Regardless of whether a geometric average of the ratio, V_n / P_n , is taken or not, this paper's results are in contrast to those of the conventional models.
6. Investors may be better off by making investment-decisions based upon the values of the ${}^eR_{mi}$ and ${}^aR_{mi}$, rather than the values of ${}^eR_{ci}$ and ${}^aR_{ci}$.

Appendix 1 provides a discussion on whether the ratio of the value-received and value-paid be subjected to geometric-averaging.

The present value of future cash flows can be defined as:

$$\begin{aligned}
 P_0 = & C_1 / (I + Y_0) \\
 & + C_2 / (I + Y_0)(I + Y_1) \\
 & + C_3 / (I + Y_0)(I + Y_1)(I + Y_2) \\
 & + \dots + C_{n-2} / (I + Y_0)(I + Y_1)(I + Y_2) + \dots + (I + Y_{n-3}) \\
 & + C_{n-1} / (I + Y_0)(I + Y_1)(I + Y_2) + \dots + (I + Y_{n-2}) \\
 & + (C_n + F) / (I + Y_0)(I + Y_1)(I + Y_2) + \dots + (I + Y_{n-1})
 \end{aligned}$$

The expected n-period value of the future cash flows is given by:

$$\begin{aligned}
 {}^eV_n = & C_1(I + Y_1)(I + Y_2)(I + Y_3) + \dots + (I + Y_{n-1}) \\
 & + C_2(I + Y_2)(I + Y_3)(I + Y_4) + \dots + (I + Y_{n-1}) \\
 & + C_3(I + Y_3)(I + Y_4)(I + Y_5) + \dots + (I + Y_{n-1}) \\
 & + \dots + C_n + F
 \end{aligned}$$

The realized n-period value of the cash flows is given by:

$$\begin{aligned}
 {}^aV_n = & C_1(I + {}^aY_1)(I + {}^aY_2)(I + {}^aY_3) + \dots + (I + {}^aY_{n-1}) \\
 & + C_2(I + {}^aY_2)(I + {}^aY_3)(I + {}^aY_4) + \dots + (I + {}^aY_{n-1}) \\
 & + C_3(I + {}^aY_3)(I + {}^aY_4)(I + {}^aY_5) + \dots + (I + {}^aY_{n-1}) \\
 & + \dots + C_n + F
 \end{aligned}$$

The expected n-period value of the asset price paid at time 0 is given by:

$${}^eP_n = P_0 [(I + Y_0)(I + Y_1)(I + Y_2) + \dots + (I + Y_{n-1})]$$

The realized n-period value of the asset price paid at time 0 is given by:

$${}^aP_n = P_0 [(I + {}^aY_0)(I + {}^aY_1)(I + {}^aY_2) + \dots + (I + {}^aY_{n-1})]$$

Proposition 1: In the proposed modified model, expected rate of return, eR_m , will be always zero. However, the realized rate of return, aR_m , may be negative, positive, or zero depending upon the subsequent changes in the market rates during the investment horizon.

Since both the numerator and the denominator of the ROR-equation are time-valued up to period n, hence the ratio of the two, $[{}^eV_n / {}^eP_n]$, will be always equal to one such that,

therefore, the first term of the expression for aP_n will be equal to the first term of the expression for aV_n ; however, the second term of the expression for aP_n will be less than the second term of the expression for aV_n , the third term of the expression for aP_n will be less than the third term of the expression for aV_n , the fourth term of the expression for aP_n will be less than the fourth term of the expression for aV_n , ..., and the n^{th} term of the expression for aP_n will be less than the n^{th} term of the expression for aV_n . Therefore,

$${}^aV_n > {}^aP_n,$$

and thus

$$[{}^aV_n / {}^aP_n] > 1$$

and

$${}^aR_m = [{}^aV_n / {}^aP_n] - 1 > 0 \text{ (QED)}$$

CHANGES IN MARKET RATES

In the proposed model, the break-even value of R is zero, at which point the investor receives what he paid for. Furthermore,

1. $R = 0$, if either Y remains constant or it changes, on average, in a neutralizing fashion
2. $R > 0$, if either Y decreases or it changes, on average, in a decreasing fashion
3. $R < 0$, if either Y increases or it changes, on average, in an increasing fashion

The proof is given below.

Accumulated coupon reinvestment effect at time h is given by:

$$\begin{aligned} (CRE)_h &= C_1(I + {}^aY_1)(I + {}^aY_2)(I + {}^aY_3) + \dots + (I + {}^aY_{h-1}) \\ &\quad + C_2(I + {}^aY_2)(I + {}^aY_3)(I + {}^aY_4) + \dots + (I + {}^aY_{h-1}) \\ &\quad + C_3(I + {}^aY_3)(I + {}^aY_4)(I + {}^aY_5) + \dots + (I + {}^aY_{h-1}) \\ &\quad + \dots + C_3(I + {}^aY_h)(I + {}^aY_{h+1})(I + {}^aY_{h+2}) + \dots + (I + {}^aY_{h-1}) \end{aligned}$$

Sale-price at time h is given by,

$$\begin{aligned} P_h^s &= C_{h+1} / (I + {}^aY_h) \\ &\quad + C_{h+2} / (I + {}^aY_h)(I + {}^aY_{h+1}) \\ &\quad + C_{h+3} / (I + {}^aY_h)(I + {}^aY_{h+1})(I + {}^aY_{h+2}) \\ &\quad + \dots + C_{n-2} / (I + {}^aY_h)(I + {}^aY_{h+1})(I + {}^aY_{h+2}) \dots (I + {}^aY_h)(I + {}^aY_{h+1})(I + {}^aY_{n-3}) \\ &\quad + C_{n-1} / (I + {}^aY_0)(I + {}^aY_1)(I + {}^aY_2) + \dots + (I + {}^aY_{n-2}) \\ &\quad + (C_n + F) / (I + {}^aY_0)(I + {}^aY_1)(I + {}^aY_2) + \dots + (I + {}^aY_{n-1}) \end{aligned}$$

Time value of the purchase price is given by,

$${}^aP_h = P_0 [(I + {}^aY_0)(I + {}^aY_1)(I + {}^aY_2) + \dots + (I + {}^aY_{h-1})]$$

The rate of return at time h is given by

$$R_{mh} = [(CRE)_h + P_h^s] / {}^aP_h - 1$$

Therefore,

$R_{mh} > 0$, if, on average, the set of the realized values of the market rates, $\{^aY_0, ^aY_1, ^aY_2, \dots, ^aY_{n-1}\}$, is greater than the set of the expected values of the market rates, $\{Y_0, Y_1, Y_2, \dots, Y_{n-1}\}$, in terms of its effect on coupon reinvestment value, sale-price value, and the time-value of the purchase price.

$R_{mh} > 0$, if, on average, the set of the realized values of the market rates, $\{^aY_0, ^aY_1, ^aY_2, \dots, ^aY_{n-1}\}$, is less than the set of the realized values of the market rates, $\{Y_0, Y_1, Y_2, \dots, Y_{n-1}\}$, in terms of its effect on coupon reinvestment value, sale-price value, and the time-value of the purchase price.

$R_{mh} = 0$, if, on average, the set of the realized values of the market rates, $\{^aY_0, ^aY_1, ^aY_2, \dots, ^aY_{n-1}\}$, is equal to the set of expected values of the market rates, $\{Y_0, Y_1, Y_2, \dots, Y_{n-1}\}$, in terms of its effect on coupon reinvestment value, sale-price value, and the time-value of the purchase price.

FLUCTUATIONS IN R DUE TO CHANGES IN Y: NUMERICAL ILLUSTRATIONS

In this section, the author proves, through numerical examples, the propositions stated in section 3.

Proposition 2: If the actual market rates are equal to the expected rates, then $^aR_m = 0$ (proof through a numerical example):

$$N = 3, C = 100, F = 1,000$$

$$Y_0 = 0.10, Y_1 = 0.08, Y_2 = 0.15$$

$$^aY_0 = 0.10, ^aY_1 = 0.08, ^aY_2 = 0.15$$

$$P_0 = [100/1.1] + 100/((1.1)*(1.08)) + (100)/((1.1)*(1.08)*(1.15)) + (1000)/((1.1)*(1.08)*(1.15)) = 980.237$$

$$^eP_n = (980.237)*[(1.1)*(1.08)*(1.15)] = 1339.2$$

$$^aP_n = (980.237)*[(1.1)*(1.08)*(1.15)] = 1339.2$$

$$^eV_n = 100*(1.08)*(1.15) + (100)*(1.15) + 100 + 1000 = 1339.2$$

$$^aV_n = 100*(1.08)*(1.15) + (100)*(1.15) + 100 + 1000 = 1339.2$$

$$^eR_c = (1339.2/980.237)^{(1/3)} - 1 = .1096$$

$$^aR_c = (1339.2/980.237)^{(1/3)} - 1 = .1096$$

$$^eR_m = (1339.2/1339.2) - 1 = 0 \text{ (ROR without geometric-averaging)}$$

$$^aR_m = (1339.2/1339.2) - 1 = 0 \text{ (ROR without geometric-averaging)}$$

$$\text{old-ROR}^* = R_{c-y} = ^aR_c - ^eR_c = .1096 - .1096 = 0 \text{ (yield-adjusted } R_c)$$

If, on average, the realized market-rates are equal to their expected level, then the value of the new-ROR will be zero and so will be the value of old-ROR*.

At maturity, the actual time value of the total revenue is exactly the same as the actual time value of the total cost such that the investor receives exactly the same asset (bundle of goods and services) for which he had initially paid. A layperson will see the asset as a vehicle for maintaining the value of his/her savings; and thus the net rate of return is perceived to be zero. Under conventional model, the expected rate of return is positive (.1096) and is equal to the actual rate of return (.1096); this is due to the fact that the time value of the price paid is ignored. The yield-adjusted-old-ROR is zero.

Proposition 3: If the actual market rates are higher than the expected rates, then $^aR_m < 0$. (proof through a numerical example):

$$N = 3, C = 100, F = 1,000$$

$$Y_0 = 0.10, Y_1 = 0.08, Y_2 = 0.15$$

$$^aY_0 = 0.10, ^aY_1 = 0.09, ^aY_2 = 0.25$$

$$P_0 = [100/1.1] + 100/((1.1)*(1.08)) + (100)/((1.1)*(1.08)*(1.15)) + (1000)/((1.1)*(1.08)*(1.15)) = 980.237$$

$${}^eP_n = (980.237)*[(1.1)*(1.08)*(1.15)] = 1339.2$$

$${}^aP_n = (980.237)*[(1.1)*(1.09)*(1.25)] = 1469.13$$

$${}^eV_n = 100*(1.08)*(1.15) + (100)*(1.15) + 100 + 1000 = 1339.2$$

$${}^aV_n = 100*(1.09)*(1.25) + (100)*(1.25) + 100 + 1000 = 1361.25$$

$${}^eR_c = (1339.2/980.237) ^ (1/3) - 1 = .1096$$

$${}^aR_c = (1361.25/980.237) ^ (1/3) - 1 = .1157$$

$${}^eR_m = (1339.2/1339.2) ^ (1/3) - 1 = 0$$

$${}^aR_m = (1361.25/1469.13) - 1 = -0.07343 < 0 \text{ (ROR without geometric-averaging)}$$

$${}^aR_{gm,3} = (1361.25/1469.13) ^ (1/3) - 1 = -0.0251 < 0 \text{ (ROR with geometric-averaging)}$$

$$\text{old-ROR}^* = R_{c-y} = {}^aR_c - {}^eR_c = .1157 - .1096 = 0.0061 > 0 \text{ (yield-adjusted } R_c)$$

If the asset is held till maturity and if, on average, the realized rates are greater than their expected levels, then the value of the new-ROR will be *negative*, while that of the old-ROR* will be *positive*. Alternatively, if the market rates, on average, rise then the new-ROR will be negative, while the adjusted-old-ROR* will be positive when the asset is held till maturity. In other words, when market rates rise, then the old-model informs to investors that their investment is appreciating in value, while the new-model correctly informs the investors that their investment is, in fact, depreciating. When market rates rise, investors ought to be happy following the old-model, while they, in fact, ought to be unhappy if the ROR-analysis is done in the context of the new-model. This has important implications for investment recommendations to the clientele.

At maturity, the actual time value of the total revenue is less than the actual time value of the total cost such that the investor receives less than what he had initially paid for; the net rate of return, therefore, is negative. When market rates rise, the opportunity cost of the price paid increases. The Future value of the earnings has also increased under scenario; however, the opportunity cost of the price paid has increased by a larger magnitude than the increase in the future value of the revenues. Consequently, the rate of return is negative. Under conventional model that uses the initial bond price as the divisor, however, the net rate of return has increased to .1196 (positive). Similarly, the yield-adjusted-old-ROR is also positive.

Proposition 4: If the actual market rates are lower than the expected rates, then ${}^aR_m > 0$ (proof through a numerical example):

$$N = 3, C = 100, F = 1,000$$

$$Y_0 = 0.10, Y_1 = 0.08, Y_2 = 0.15$$

$${}^aY_0 = 0.10, {}^aY_1 = 0.03, {}^aY_2 = 0.12$$

$$P_0 = [100/1.1] + 100/((1.1)*(1.08)) + (100)/((1.1)*(1.08)*(1.15)) + (1000)/((1.1)*(1.08)*(1.15)) = 980.237$$

$${}^eP_n = (980.237)*[(1.1)*(1.08)*(1.15)] = 1339.2$$

$${}^aP_n = (980.237)*[(1.1)*(1.03)*(1.12)] = 1243.88$$

$${}^eV_n = 100*(1.08)*(1.15) + (100)*(1.15) + 100 + 1000 = 1339.2$$

$${}^aV_n = 100*(1.03)*(1.12) + (100)*(1.12) + 100 + 1000 = 1327.36$$

$${}^eR_c = (1339.2/980.237) ^ (1/3) - 1 = .1096$$

$${}^aR_c = (1327.36/980.237) ^ (1/3) - 1 = 0.106333 > 0$$

$${}^eR_m = (1339.2/1339.2) - 1 = 0$$

$${}^aR_m = (1327.36/1243.88) - 1 = 0.067113 > 0 \text{ (ROR without geometric-averaging)}$$

$$R_{gm,3} = (1327.36/1243.88) ^ (1/3) - 1 = 0.021888 > 0 \text{ (ROR with geometric-averaging)}$$

$$R_{c-y} = {}^aR_c - {}^eR_c = 0.106333 - 0.1096 = -0.00327 < 0 \text{ (yield-adjusted } R_c)$$

If the asset is held till maturity and if, on average, the realized rates are less than their expected levels, then the value of the new-ROR will be *positive*, while that of the old-ROR* will be *negative*. Alternatively, if the market rates, on average, fall then the new-ROR will be positive, while the adjusted-old-ROR* will be negative when the asset is held till maturity. In other words, when market rates rise, then the old-model informs to investors that their

investment is depreciating in value, while the new-model correctly informs the investors that their investment is, in fact, appreciating. When market rates fall, investors ought to be unhappy following the old-model, while they, in fact, ought to be happy if the ROR-analysis is done in the context of the new-model. This has important implications for investment recommendations to the clientele.

At maturity, the actual time value of the total revenue is greater than the actual time value of the total cost such that the investor receives more than what he had initially paid for; the net rate of return, therefore, is positive. When market rates decrease, the opportunity cost of the price paid decreases. The Future value of the earnings also decreases, however, the opportunity cost of the price paid has decreased by a larger magnitude than the decrease in the future value of the revenues. Consequently, the rate of return is positive. Under conventional models, that use the initial bond price as the divisor, however, the net rate of return has decreased from .1096 (under constant Y) to .1063 (under decreased Y).

CONCLUDING REMARKS

The time-valued asset-prices and asset-earnings become instant-prices paid and instant-earnings received for the entire holding period, and hence the ratio of the value-received and value-paid need not be subjected to geometric-averaging, which is in contrast to the conventional models, where such an averaging is a necessity. The paper also considers geometric-averaging of the ratio by treating the value-received and value-paid as the total revenues and total costs respectively for the entire holding period. The paper's results are valid irrespective of whether or not the ratio of the value-received and value-paid is subjected to geometric-averaging. Under the geometric-averaging of the ratio of the value-received and value-paid, if market rates rise, then the longer the bond is held the better, since the negative-value of the ROR diminishes as holding period increases; while if market rates fall, then the shorter-period the bond is held the better, since the positive-value of the ROR increases as holding period decreases.

The fundamental difference between the proposed model and the conventional model lies in the denominator of the ROR expression, where the proposed model uses the reinvestment value of the asset-price, while conventional models use the initial price paid at time 0. When market rates rise (fall), then the time-value of the asset-price-paid increases (decreases) by a larger (smaller) magnitude than the asset-earnings-received, thereby making the ROR negative (positive).

The paper defines the appreciation and depreciation of the investment value in terms of the positivity and negativity of the rate of return respectively. If market rates rise and the asset is held till maturity, then the new-ROR is negative, implying a depreciation in investment value, while the old-ROR* is positive, implying an appreciation in investment value. Similarly, if market rates fall and the asset is held till maturity, then the new-ROR is positive, implying an appreciation in investment value, while the old-ROR* is negative, implying an appreciation in investment value. Only when the market rates remain constant during the investment horizon that the model's yield identical results. The new-ROR of this paper draws diametrically opposite conclusions to those of the conventional-ROR. Individuals reveal an overwhelming preference for the new-ROR due to the ease of information and clarity.

APPENDIX 1

Geometric Averaging In ROR

1. The ratio (value-received/value-paid) should not be subjected to geometric-averaging (because it is viewed as the ratio of instant sale-price and instant purchase-price at time H , the holding-time)
2. The ratio (value-received/value-paid) should be subjected to geometric-averaging (because it is viewed as the ratio of total-revenue and total-costs at time H , the holding time)

THE RATIO (VALUE-RECEIVED / VALUE-PAID) SHOULD NOT BE SUBJECTED TO GEOMETRIC-AVERAGING

Using the instant price-received at time n and instant price-paid at time n , the ratio of value-received and value-paid should not be n^{th} rooted.

At each point in time, R is determined by taking the ratio between the value-received and value-paid; as time progresses, of course more coupons will be received, but at the same time, equi-proportionately, more costs will also

be incurred (in the form of higher time-value of the price paid). The changes in the costs as well as revenues will occur in an R-neutralizing fashion. With time, the revenues will rise, but, equi-proportionately, so will the costs to keep the rate of return constant at all times, If the market rate changes during the first period and remains at that level throughout the asset-life, then the rate of return becomes time-neutral (independent of the holding time) such that the returns will be the same regardless of whether the asset is disposed off instantly or at a later date. The investor would not gain by holding the asset for a longer period of time. For example, if market rate decreases (to cause R to be 41%) a few moments later on day 0 and is expected to remain that way until maturity, then it does not matter whether the asset is sold immediately or it is held until maturity; the investor will receive only 41% return under all circumstances (on day 0, he receives 41% on an investment of P_0 ; at time 1, he receives 41% on an investment of $P_0(I+Y_{new})$; at time 2, he receives 41% on an investment of $P_0(I+Y_{new})^2$; at time 3, he receives 41% on an investment of $P_0(I+Y_{new})^3$; at time n, he receives 41% on an investment of $P_0(I+Y_{new})^n$; and so on. In other words, investor invests less during first year in the amount, P_0 , and gets a 41% return on it, while during the nth year, he invests more in the amount, $P_0(I+Y_{new})^n$, and still gets a return of only 41%. Then why wait until the nth year to invest a larger sum, why not invest a sum in the amount, $P_0(I+Y_{new})^n$, during first year to get a return of 41%. If the investor is confident that the market rate will decrease to yield a return of 41% and it will remain at that level until maturity, then he need not wait but he should cash in the return of 41% instantly, as waiting for another day will not yield any higher returns.

However, if the investor's expectations with regard to the constancy of the decreased market rate is not realized, then he may be worse off (or better off) by selling the bond earlier depending upon in what direction (rise or fall) he was incorrect in his expectations. In particular, if he cashes in the profits during first year under the expectations of a constant market rate, and the market rate subsequently further falls during second year, then his return would have been higher had he waited until second year.

THE RATIO (VALUE-RECEIVED/VALUE-PAID) SHOULD BE SUBJECTED TO GEOMETRIC-AVERAGING

Using the total revenue and total cost principle to determine the rate of profit in n-periods, the ratio of revenue and cost be nth rooted.

$$\begin{array}{ccccccc} \hline & | & & | & & | & & | & & | \\ 0 & & 1 & & 2 & & 3 & & & n \\ \hline P_0 & & P_0(I+Y_0) & & P_0(I+Y_0)(I+Y_1) & & P_0(I+Y_0)(I+Y_1)(I+Y_2) & & & P_0(I+Y_0)(I+Y_1)\dots(I+Y_{n-1}) \\ \hline \end{array}$$

$$TC = FC + VC$$

$$FC = P_0$$

$$VC = VC_1 + VC_2 + VC_3 + \dots + VC_n$$

$$\begin{aligned} VC &= [Y_0P_0] + [Y_1P_0(I+Y_0)] + [Y_2P_0(I+Y_0)(I+Y_1)] + \dots + [Y_{n-1}P_0(I+Y_0)(I+Y_1)\dots(I+Y_{n-2})] \\ &= P_0\{[Y_0] + [Y_1(I+Y_0)] + [Y_2(I+Y_0)(I+Y_1)] + \dots + [Y_{n-1}(I+Y_0)(I+Y_1)\dots(I+Y_{n-2})]\} \end{aligned}$$

thus,

$$TC = P_0 + P_0\{[Y_0] + [Y_1(I+Y_0)] + [Y_2(I+Y_0)(I+Y_1)] + \dots + [Y_{n-1}(I+Y_0)(I+Y_1)\dots(I+Y_{n-2})]\}$$

where,

$$VC_1 = [P_0(I+Y_0)] - [P_0] = Y_0P_0$$

$$VC_2 = [P_0(I+Y_0)(I+Y_1)] - [P_0(I+Y_0)] = Y_1P_0(I+Y_0)$$

$$VC_3 = [P_0(I+Y_0)(I+Y_1)(I+Y_2)] - [P_0(I+Y_0)(I+Y_1)] = Y_2P_0(I+Y_0)(I+Y_1)$$

$$VC_n = [P_0(I+Y_0)(I+Y_1)\dots(I+Y_{n-2})(I+Y_{n-1})] - [P_0(I+Y_0)(I+Y_1)\dots(I+Y_{n-2})]$$

$$= Y_{n-1}P_0(I+Y_0)(I+Y_1)\dots(I+Y_{n-2})$$

The total accumulated revenue in n -years = $TR = {}^aV_n$

The total accumulated cost in n -years = TC

Therefore, the rate of return should be n^{th} rooted:

$$R_n = (TR/TC)^{(1/n)} - 1$$

WITHOUT GEOMETRIC-AVERAGING OF THE RATIO OF VALUE-RECEIVED AND VALUE-PAID

Within the purview of this paper's model, if market rate increases during first year and remains at that level until maturity, then the investor's rate of return will be a constant negative number for any holding period (H) relative to duration (D), (i.e., irrespective of whether $H > D$, $H < D$, $H = D$, $H = 0$, or $H = n$). This is in stark contrast with the conventional results, where if market increases then $R > Y$ for $H > D$, $R < Y$ for $H < D$, and $R = Y$ for $H = D$.

Within the purview of this paper's model, if market rate decreases during first year and remains at that level until maturity, then the investor's rate of return will be a constant positive number for any holding period (irrespective of whether $H > D$, $H < D$, $H = D$, $H = 0$, or $H = n$). This is in stark contrast with the conventional results, where if market decreases then $R < Y$ for $H > D$; $R > Y$ for $H < D$; and $R = Y$ for $H = D$.

In this paper's model, if market rate does not change at all during the life of the asset, then $R = 0$ for any holding period, while under conventional models, $R = Y$ for any holding period.

UNDER GEOMETRIC-AVERAGING OF THE RATIO OF VALUE-RECEIVED AND VALUE-PAID

If Y remains constant throughout the life of the bond, then the rate of return under the proposed framework will be 0, while under conventional framework it will be equal to the yield to maturity. This statement is true regardless of the distance between duration and the holding period (i.e., irrespective of whether $H = D$, $H > D$, or $H < D$).

If Y increases, then under conventional framework, $R = Y$ at duration, $R > Y$ past the duration, and $R < Y$ before the duration. Under the proposed framework, $R < 0$ if Y increases, and the negative rate of return diminishes in value as the holding period increases and approaches maturity. Thus both models appear to produce similar results: if Y increases, then the longer the asset is held the better.

If Y decreases, then under the conventional framework, $R = Y$ at duration, $R < Y$ past the duration, and $R > Y$ before the duration. Under the proposed framework, $R > 0$ if Y decreases, and the positive rate of return diminishes in value as the holding period increases and approaches maturity. Thus both models appear to produce similar results: if Y decreases, then the shorter the holding period, the better (i.e., the asset should be discarded as quickly as possible).

REFERENCES

- [1] Anderson, Torben G., and Tim Bollerslev, "Heterogeneous Information Arrivals and Return Volatility Dynamics: Uncovering the Long-Run in High Frequency Returns," *Journal of Finance*, Vol. 52(3), July 1997, pp. 975-1006.
- [2] Anderson, Torben G., "Return Volatility and Trading Volume: an Information Flow Interpretation of Stochastic Volatility," *Journal of Finance*, Vol. 51, 1996, pp. 169-204.
- [3] Abel, Andrew B., and Oliver J. Blanchard, "The Present Value of Profits and Cyclical Movements in Investment," *Econometrica*, Vol. 52, 1986, pp. 249-273.
- [4] Bodie, Z., A Kane, and A. Marcus, *Essentials of Investments*, New York: Irwin/McGraw-Hill, Inc., 1998.

- [5] Capettini, R., R. Grimlund, and H. Toole, "Comment: The Unique Real Internal Rate of Returns," *Journal of Financial and Quantitative Analysis*, December 1979.
 - [6] Cox, J.C., J.E. Ingersoll, and S.A. Ross, "A Theory of the Term Structure of Interest Rates," *Econometrica*, Vol. 53, March 1985, pp. 385-407.
 - [7] Daniel, K. and Sheridan Titman, "Evidence on the Characteristics of Cross-Sectional Variation in Stock Returns," *Journal of Finance*, Vol. 52(1), March 1997, pp. 1-34.
 - [8] Dorfman, R., "The Meaning of the Internal Rate of Return," *Journal of Finance*, Vol. 36, December 1981, pp. 1010-1023.
 - [9] Fama, Eugene F., and Kenneth R. French, "The Cross-Section of Stock Returns," *Journal of Finance*, Vol. 47, 1992, pp. 427-465.
 - [10] Fama, Eugene F., and Kenneth R. French, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Finance*, Vol. 33, 1993, pp. 3-56.
 - [11] Fama, Eugene F., and Kenneth R. French, "Size and Book to Market Factors in Earnings and Returns," *Journal of Finance*, Vol. 50, 1995, pp. 131-156.
 - [12] Fama, Eugene F., and Kenneth R. French, "Multifactor Explanation of Asset Market Anomalies," *Journal of Finance*, Vol. 51, 1996, pp. 505-84.
 - [13] Gilchrist, Simon, and Charles Himmelberg, "Evidence on the Role of Cash Flow for Investment," *Journal of Monetary Economics*, Vol. 36, 1996, pp. 541-572.
 - [14] Hearth, D. and J. Zaima, *Contemporary Investments*, Fortworth: Dryden Press, 1998.
 - [15] Hougén, R.A., *Modern Investment Theory*, Englewood Cliffs, NJ: Prentice-Hall, 1997.
 - [16] Keown, Arthur J., *Personal Finance: Turning Money into Wealth*, Englewood Cliffs, NJ: Prentice-Hall, 1998.
 - [17] Lamant, Owen, "Cash Flow and Investment: Evidence from Internal Capital Markets," *Journal of Finance*, Vol 52(1), March 1997, pp. 83-110.
 - [18] Marshall, David, "Inflation and Asset Returns in a Monetary Economy," *Journal of Finance*, Vol. 47, 1992, pp. 1315-1342.
 - [19] Merton, Robert C., "On Estimating the Expected Return on the Market," *Journal of Financial Economics*, Vol. 8, 1980, pp. 323-361.
 - [20] Roma, A. and W. Torous, "The Cyclical Behavior of Interest Rates," *Journal of Finance*, Vol. 52(4), pp. 1519-1542.
 - [21] Sias, R.W. and Laura T. Starks, "Institutions and Individuals at the Turn-of-the-Year," *Journal of Finance*, Vol. 52(4), pp. 1543-1562.
 - [22] Snow, Karl N., "Diagnosing Asset Pricing Models Using the Distribution of Asset Returns," *Journal of Finance*, July 1991, pp. 955-984.
 - [23] Strong, Robert, *Practical Investment Management*, Cincinnati, OH: South-Western College Publishing Company, 1997.
 - [24] Stulz, Rene M., "Asset Pricing and Expected Inflation," *Journal of Finance*, Vol. 41(1), 1986, pp. 209-224
 - [25] Thorbecke, Willem, "On Stock Market Returns and Monetary Policy," *Journal of Finance*, Vol 52(2), June 1997, pp. 635-654.
-